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The Speed of Light in a Moving Medium

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Abstract

The Fizeau experiment is analysed in terms of special relativity on the hypothesis that light in a material medium is alternately emitted and absorbed. It is found that if dispersion is neglected this 'extinction' theory leads to the same result as the more usual theory, contrary to a recent conclusion of Ockert. The theories differ in their predictions when dispersion is included, and the results of an experiment by Zeeman are shown to favour the usual theory.

1. *Introduction*

Ockert (1968) has recently discussed the Fizeau experiment, involving the transmission of light along a stream of water of refractive index n and speed v, in terms of the extinction theory of light transmission, i.e. the theory that the light is alternately absorbed and re-emitted by atoms, and between emission and absorption travels in the form of photons with speed c. He showed that the experimental result

$$u = c/n + v(1 - 1/n^2) \text{ to first order in } (v/c) \tag{1.1}$$

where u is the average speed of the light in the laboratory frame of reference, and c is the speed of light *in vacuo*, is inconsistent with the Ritz theory of light propagation. He then argued that the result must equally be inconsistent with special relativity, since both this and the Ritz theory state that the velocity of a photon is c relative to its source (in this case, a water molecule). There is, however, a crucial difference between the two theories. In the Ritz theory, addition of velocities follows the Newtonian rule, so that the velocity of an emitted photon in the laboratory frame of reference is $(c + v)$, and this fact is used by Ockert. In special relativity the velocity of the photon is c in all frames, so that Ockert's argument does not apply. It is the purpose of this note to show that special relativity together with the extinction theory in fact yields (1.1), as does the more usual special relativistic argument in which light is considered as a signal of velocity c/n relative to the water. However, as will be shown below, the extinction theory gives a result differing from the more usual theory when dispersion is taken into account, and the balance of experimental evidence appears to be in favour of the usual theory.

2. *Derivation of the Main Result*

We define two frames of reference –S, the laboratory frame of reference, and S', a frame moving with the stream of water and therefore moving with velocity v along the x-axis of S. All events may be considered to be on the x-axis of both frames. For the sake of simplicity we consider only three events in the life of a photon:

A. The photon is emitted at:
space and time coordinates (0,0) in S and (0,0) in S'

B. The photon is absorbed at:
space and time coordinates (x_B, t_B) in S and (x_B', t_B') in S'

C. The photon is re-emitted at:
space and time coordinates (x_c, t_c) in S and (x_c', t_c') in S'.

In reality the sequence ABC will be repeated many times. We shall suppose that the space and time separations of the events have their average values, so that if u and u' are the average speeds of the photon in the two frames of reference,

$$u = x_c/t_c, \qquad u' = x_c'/t_c' \qquad \text{(2.1a, 1.2b)}$$

The basic equations required are as follows:

The Lorentz transformation: Of this, all that is needed is

$$t_B = \beta\left(t_B' + \frac{v}{c^2}x_B'\right), \qquad t_c = \beta\left(t_c' + \frac{v}{c^2}x_c'\right) \qquad \text{(2.2a, 2.2b)}$$

where, as usual, $\beta \equiv [1 - (v^2/c^2)]^{-1/2}$.

The speed of light is c in both frames: This implies

$$x_B = ct_B, \qquad x_B' = ct_B' \qquad \text{(2.3a, 2.3b)}$$

The speed of the water is v in the laboratory frame S, and zero in the frame S′ which moves with the water: This implies

$$x_c = x_B + v(t_c - t_B), \qquad x_c' = x_B' \qquad \text{(2.4a, 2.4b)}$$

The refractive index of the water, n, is defined by

$$\frac{c}{n} = u' \qquad \text{(2.5)}$$

First consider (2.5) in conjunction with (2.1b), (2.4b) and (2.3b). We find

$$\frac{c}{n} = \frac{x_c'}{t_c'} = \frac{x_B'}{t_c'} = \frac{ct_B'}{t_c'}, \qquad t_c' = nt_B' \qquad \text{(2.6)}$$

Substituting this in (2.2) and again using (2.3b) and (2.4b) gives

$$t_B = \beta\left(t_B' + \frac{v}{c^2}.ct_B'\right) = \beta\left(1 + \frac{v}{c}\right)t_B' \tag{2.7a}$$

$$t_c = \beta\left(nt_B' + \frac{v}{c^2}.ct_B'\right) = \beta\left(n + \frac{v}{c}\right)t_B' \tag{2.7b}$$

so that

$$t_c - t_B = \beta(n-1)\,t_B' \tag{2.8}$$

Substituting (2.8) in (2.4a), and also using (2.7a) and (2.3a), we find

$$\begin{aligned} x_c &= x_B + v(t_c - t_B) = ct_B + v\beta(n-1)\,t_B' \\ &= \left[c\beta\left(1 + \frac{v}{c}\right) + \beta(n-1)\,v\right]t_B' = \beta(c + nv)\,t_B' \end{aligned} \tag{2.9}$$

Finally, substituting (2.9) and (2.7b) in (2.1a) gives

$$\begin{aligned} u &= \frac{x_c}{t_c} = \frac{\beta(c+nv)\,t_B'}{\beta\left(n + \frac{v}{c}\right)t_B'} = \frac{c}{n}\left\{\frac{1 + (nv/c)}{1 + (v/nc)}\right\} \\ &= \frac{c}{n} + v(1 - 1/n^2) - \frac{v^2}{c}(1/n - 1/n^3) + \ldots \end{aligned} \tag{2.10}$$

so that (1.1) is obtained to first order in v/c. In the usual relativistic theory, the exact result (2.10) is obtained immediately from the velocity addition formula

$$u = \frac{u' + v}{1 + (u'v/c^2)}, \qquad u' \equiv \frac{c}{n} \tag{2.11}$$

The above argument reveals (2.11) as an elegant short cut if u' is interpreted as an average velocity resulting from emission and absorption events in the fluid.

3. *The Effects of Dispersion*

In deriving (2.10) the refractive index n has been defined in terms of the average speed of light u' in the moving reference frame S'. Since n is a function of frequency, $n = n(\nu)$, and since the frequency ν_m' of the light measured in S' will differ from the frequency ν_0 measured in the laboratory, an allowance for dispersion must be made in calculating the theoretical value of (2.10) or of (1.1). It is convenient to define a coefficient k by

$$u = \frac{c}{n_0} + kv, \qquad n_0 \equiv n(\nu_0) \tag{3.1}$$

where, to first order in v/c, k is independent of v. Landsberg (1961) has derived a general expression for k, considering light in the usual way as a signal of velocity c/n in the medium. He finds

$$k = 1 - \frac{1}{n_0{}^2} + \frac{\nu_0}{n_0}\left(\frac{dn}{d\nu}\right)_{\nu 0} \tag{3.2a}$$

$$k = 1 - \frac{1}{n_0{}^2} + \frac{\nu_0}{n_0{}^2}\left(\frac{dn}{d\nu}\right)_0 \tag{3.2b}$$

where (3.2a) applies to a Fizeau drag experiment and (3.2b) to an experiment in which the medium and its container move together with speed v (e.g. a moving glass block).

Following Landsberg, we suppose that the Doppler effect causes a difference in frequency which can be expressed to first order as

$$\nu_m{}' = \nu_0(1 - g) \tag{3.3}$$

Then expanding $n(\nu)$ in a Taylor series about ν_0, substituting into (2.10) and retaining only first order terms in g gives

$$k = 1 - \frac{1}{n_0{}^2} + \frac{gc}{v}.\frac{\nu_0}{n_0{}^2}\left(\frac{dn}{d\nu}\right)_{\nu 0} \tag{3.4}$$

Now according to the extinction theory, light travels as a signal of speed c (between emission and absorption) in the moving medium as well as *in vacuo*, so that the special relativistic Doppler effect formula for a signal of speed w,

$$\nu_m{}' = \frac{\nu_0[1 - (v^2/c^2)]^{1/2}}{1 + (v/w)} \tag{3.5}$$

can be applied with $w = c$. This gives

$$\nu_m{}' = \nu_0\left[\frac{1 - (v/c)}{1 + (v/c)}\right]^{1/2} \doteqdot \nu_0\left(1 - \frac{v}{c}\right), \qquad g = v/c \tag{3.6}$$

so that for the extinction theory (3.4) becomes (3.2b). This value of k will be called k_{ext}. This argument holds for either the Fizeau or the moving block experiments. Comparison with (3.2a, b) shows that the moving block experiment does not distinguish between the extinction theory and the conventional theory. For the Fizeau experiment, however, the conventional theory predicts (3.2a). This value of k will be called k_{con}. The difference between the two calculated values is then

$$\Delta k = k_{\text{con}} - k_{\text{ext}} = \frac{\nu_0}{n_0}\left(1 - \frac{1}{n_0}\right)\left(\frac{dn}{d\nu}\right)_{\nu 0} \tag{3.7}$$

4. *Comparison with Experiments*

It appears that the most recent experimental evidence suitable for an evaluation of (3.7) is that of Zeeman (1914, 1916), who carried out a Fizeau experiment with water at four different wavelengths and compared the observed values of k with those calculated on the conventional theory. The results of applying (3.7) are summarised in Table 1.

TABLE 1

Wavelength	$k_{observed}$	k_{con}	Δk	k_{ext}
λ (Å)				
4500	0·465	0·464	0·005	0·459
4580	0·463	0·463	0·005	0·458
5461	0·451	0·454	0·004	0·450
6870	0·445	0·447	0·002	0·445

It will be observed that k_{ext} is systematically lower than $k_{observed}$, with a mean error of −0·003, while k_{con} has a mean error of +0·001. The experimental evidence thus supports the conventional rather than the extinction theory.

Ockert (1968) explains (1.1) in terms of an extinction theory in which Newtonian velocity addition is used and an ether is assumed to exist and to be strongly coupled to conducting substances, so that in the Fizeau experiment, in which the tube carrying the water was conducting, the ether would be at rest in the laboratory frame. This theory will be referred to as the convective ether theory. In this theory, (3.6) is replaced by the Newtonian Doppler effect formula $\nu_m' = \nu_0[1-(v/c)]$, and the identification of g as v/c is unchanged. It follows that the evidence of Zeeman quoted above, which tends to discredit the special relativistic extinction theory, also tends to discredit Ockert's convective ether theory, since both theories predict the same first-order effects.

If further experiments were to confirm (3.2b) for the Fizeau experiment, thus supporting the idea of an extinction theory, one might consider Ockert's proposal of repeating the Fizeau experiment with a conducting medium flowing in a non-conducting tube. The function of the experiment would not however be, as suggested by Ockert, merely to confirm or deny the convective ether theory (which predicts $k \doteq 1$), but to decide between this and the special relativistic extinction theory which continues to predict (3.2b).

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